The Berry Phase and Monopoles
in Non-Abelian Gauge Theories.

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Abstract

We consider the quantum mechanical notion of the geometrical (Berry) phase in SU(2) gauge theory, both in the continuum and on the lattice. It is shown that in the coherent state basis eigenvalues of the Wilson loop operator naturally decompose into the geometrical and dynamical phase factors. Moreover, for each Wilson loop there is a unique choice of U(1) gauge rotations which do not change the value of the Berry phase. Determining this U(1) locally in terms of infinitesimal Wilson loops we define monopole-like defects and study their properties in numerical simulations on the lattice. The construction is gauge dependent, as is common for all known definitions of monopoles. We argue that for physical applications the use of the Lorenz gauge is most appropriate. And, indeed, the constructed monopoles have the correct continuum limit in this gauge. Physical consequences are briefly discussed.

Introduction

In his seminal paper [1], Berry found that a quantum state acquires an extra phase factor when the Hamiltonian of the system depends on external parameters and the state is adiabatically transported along a closed path in the parameter space. The additional phase factor depends only on this path and reflects the geometry of the space of parameters. The mathematical construction behind the geometrical Berry phase was discovered by Simon [2], who showed that the adiabatic evolution of the quantum system naturally defines a line bundle structure over the parameter space. The geometrical phase is the holonomy of the U(1) connexion and cannot be removed by the global redefinition of the wave function when the bundle is non trivial. Moreover, the adiabatic approximation can be avoided and in fact any evolution of a quantum system naturally gives rise to the geometrical phase factor [3] (for a detailed discussion of the non adiabatic Berry phase see [4] and references therein). The Berry phase is again the holonomy of the U(1) connexion, but the base space of the bundle in question is now the space of all physical states. A unified consideration of the adiabatic and non adiabatic geometrical phases was presented in [5, 6], where it was shown that these two, a priori different phase factors are actually related by the classification theorem for vector bundles. Note that the geometrical phase in quantum mechanics is ultimately related to the construction of the Abelian monopole since this is the only topologically non trivial object which arises when the structure group is U(1).

In this paper we apply the notion of the Berry phase in the SU(2) pure gauge theory[1] and, in particular, in the lattice formulation of the SU(2) gluodynamics. It is known that the geometrical phase construction is useful in chiral gauge theories (see, e.g., [7]). Here we consider the Wilson loop operator in pure gluodynamics from the quantum mechanical point of view. Indeed, the P–exponent of the gauge potential is defined as a solution of the first order differential equation, which is very similar to the Schrödinger equation. Therefore the Wilson loop may be interpreted as the evolution operator of a quantum mechanical system, where the role of the time-dependent Hamiltonian is played by the gauge potential. Since the Wilson loop is an operator in the representation space of the SU(2) group, the space of the physical states is naturally described by the generalized coherent states [8], which parameterize the coset space $SU(2)/U(1)$. We show that in the coherent state basis eigenvalues of the Wilson loop naturally decompose into dynamical and geometrical phase factors. For an infinitesimal Wilson loop we obtain the geometrical phase in terms of the continuum gauge potentials and argue that it reflects the non triviality of the Hopf bundle $SU(2) \rightarrow SU(2)/U(1) = S^2$.

Our considerations allow to define monopole-like defects in the SU(2) gluodynamics, which are different from the Abelian monopoles considered so far (see [9] for a review). Indeed, the usual definition of Abelian monopoles implies a particular partial gauge fixing which leaves a U(1) subgroup unfixed and the monopoles are defined with respect to this remaining U(1). While in the standard treatment the U(1) is universally defined for all

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[1] The SU(2) gauge group is taken for simplicity, the generalization to the SU(N) group is straightforward.

the monopoles, in our construction the relevant U(1) subgroup is determined locally for each infinitesimal Wilson loop or, in the lattice formulation, for each plaquette. Therefore, the monopoles constructed are strictly speaking non-Abelian objects. The existence of the $U(1)$ group inherent for each plaquette was in fact noticed in Ref. [10] and is central for the construction developed here.

In particular, the consideration of the geometrical phase gives a new method to calculate the trace of the Wilson loop, which is periodic function of the phase angle, $\frac{1}{2}\operatorname{Tr} W = \cos\varphi$. We propose a method to calculate the angle φ without the restriction $\varphi \in [-\pi; \pi]$ and introduce the natural decomposition:

$$\varphi = \varphi \bmod 2\pi + 2\pi k, \qquad k \in Z. \tag{1}$$

The Eq. (1), when applied to the elementary plaquette, gives a lattice analog of the Dirac strings associated with monopoles we have constructed. Therefore it becomes possible to consider the non-perturbative dynamics of these defects on the lattice.

An evident problem, which can be understood already in terms of the quantum mechanical analogy, is that the monopoles constructed in this way are not SU(2) gauge invariant. Following Ref. [11] we argue that the most appropriate gauge is the Lorenz gauge, in which gauge dependent singularities of the gauge fields are maximally suppressed. By numerical calculations we show that the monopoles we have constructed are physical objects. In particular, the density of these monopoles is shown to scale correctly towards the continuum limit. We show that the asymmetry of the monopole currents behaves as an order parameter of the deconfinement phase transition. Finally, it is shown that these monopoles are locally correlated with the non-Abelian action density, like the monopoles in the Maximal Abelian gauge, Ref. [12].

The outline of the paper is as follows. In Section 1 we briefly review some well known facts about the adiabatic and non adiabatic Berry phases in quantum mechanics. In Section 2 we consider the Wilson loop operator in the SU(2) gluodynamics. In Section 3 the SU(2) lattice gauge theory is considered. In particular, we present an explicit calculation of the Berry phase for an arbitrary Wilson loop on the lattice. In Section 4 we discuss the results of the numerical simulations. Our conclusions are summarized in the last section.

1. Berry Phase in Quantum Mechanics

Following the original paper of Berry [1], consider a hermitian Hamiltonian $H(t) = H(\lambda_i(t))$ which is time-dependent through a set of parameters $\lambda_i(t)$, $i = 1, ..., n$. We are interested in evolution of the quantum mechanical system $|\psi(t)\rangle$ governed by the Schrödinger equation:

$$i\partial_t |\psi(t)\rangle = H(\lambda(t)) |\psi(t)\rangle \tag{2}$$

on the time interval $[0; T]$. Suppose, that for any fixed $t \in [0; T]$ the spectrum of H is non-degenerate:

$$H(\lambda) |n(\lambda)\rangle = E_n(\lambda) |n(\lambda)\rangle \qquad E_n \neq E_m \tag{3}$$

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and that the time evolution of the parameters $\lambda(t)$ is slow, such that there are no induced transitions between energy levels. Then the adiabatic approximation applies and if the quantum mechanical system at $t = 0$ is described by an eigenstate of Hamiltonian $|\psi(0)\rangle = |n(\lambda(0))\rangle$, the whole evolution reduces to a phase factor:

$$|\psi(t)\rangle = e^{i\varphi(t)} |n(\lambda(t))\rangle. \tag{4}$$

Moreover, the phase factor φ acquired by the state $|\psi(t)\rangle$ as a result of the adiabatic evolution consists of two parts [1]:

$$|\psi(t)\rangle = e^{i\gamma_n(t)} \exp\{-i \int\limits_0^t E_n(\tau)d\tau\} |n(\lambda(t))\rangle, \tag{5}$$

where $\delta = -\int E_n$ is the dynamical phase while the additional phase factor γ_n is known as the adiabatic Berry phase. The latter can be found by substituting (5) into the Schrödinger equation (2):

$$\partial_t \gamma_n = i\langle n(\lambda) | \frac{\partial}{\partial \vec{\lambda}} | n(\lambda)\rangle \, \partial_t \vec{\lambda}. \tag{6}$$

Consider now the parameters $\lambda_i(t)$ which are T-periodic, $\lambda_i(0) = \lambda_i(T)$, and span a closed curve $\mathcal{C}$ in the parameter space. Then in the adiabatic approximation the initial and final state vectors differ only by a phase

$$|\psi(T)\rangle = e^{i\varphi(T)} |\psi(0)\rangle. \tag{7}$$

A state vector which obeys Eq. (7) is called the cyclic state. The Berry phase $\gamma_n(T)$ acquired by the cyclic state $|\psi\rangle$ may be represented in terms of the "gauge potential":

$$A_i = i\langle n(\lambda) | \frac{\partial}{\partial \lambda_i} | n(\lambda)\rangle, \tag{8}$$

$$\gamma_n(T) = \int\limits_{\mathcal{C}} A = \int\limits_{\mathcal{S_C}} F, \qquad F = dA, \tag{9}$$

where $\mathcal{S_C}$ is an arbitrary surface in the parameter space bounded by the contour $\mathcal{C}$. In the form (8,9) the geometrical nature of the Berry phase is almost evident. For this reason the Berry phase factor is also called geometrical phase. Indeed, the rich mathematical structure behind (8, 9) was discovered by Simon [2] and further elaborated in a number of papers [3, 5, 6], [13]–[16]. In particular, it was shown in [3] that the adiabaticity requirement is in fact unnecessary. Moreover, the precise form of the Hamiltonian is not important either. The geometrical phase factor depends only on the path $\mathcal{C}$ in the space of the physical states followed by the vector $|\psi\rangle$ and is naturally associated with any quantum evolution. For self-consistency we briefly review the essentials of the construction.

Consider unitary evolution of a quantum mechanical system described by a state vector $|\psi(t)\rangle$, where $|\psi\rangle$ is an element of an $N + 1$-dimensional complex vector space $\mathbb{C}^{N+1}$ with finite N, for simplicity. In terms of the complex coordinates $\{z_0, ..., z_N\}$ the state vector is $|\psi(t)\rangle = \{z_0(t), ..., z_N(t)\}$. Since the unitary evolution preserves the norm, the normalization condition $\langle\psi|\psi\rangle = 1$ defines a $2N+1$-dimensional sphere $S^{2N+1} \in \mathbb{C}^{N+1}$ on which the evolution takes place. The physical states are not given by normalized vectors in $\mathbb{C}^{N+1}$, since $|\psi\rangle$ and $|\psi'\rangle$ define the same physical state if they differ by a phase $|\psi\rangle = e^{i\alpha}|\psi'\rangle$. Therefore, the set of physical states is the N-dimensional projective space which is a Kählerian manifold:

$$\mathbb{C}P^N = S^{2N+1}/U(1).\tag{10}$$

The quantum mechanical evolution is given by the Schrödinger equation (2), where the Hamiltonian depends implicitly on time through the set of the parameters $\lambda_i(t)$, $\lambda_i(0) = \lambda_i(T)$. We are interested in a cyclic vector $|\psi(t)\rangle$ which returns to the same physical state $|\psi(T)\rangle = e^{i\varphi(T)}|\psi(0)\rangle$ after evolving along a closed path $\mathcal{C} \in \mathbb{C}P^N$. The phase $\varphi(T)$ is the total phase acquired by the cyclic vector. Moreover, we introduce a single-valued vector $|\tilde{\psi}(t)\rangle$ which differs from $|\psi(t)\rangle$ by a phase and satisfies the condition:

$$|\tilde{\psi}(T)\rangle = |\tilde{\psi}(0)\rangle.\tag{11}$$

Note that Eq. (11) does not define the $|\tilde{\psi}\rangle$ uniquely: for a given $|\tilde{\psi}\rangle$, vector $e^{i\theta}|\tilde{\psi}\rangle$ is also single-valued provided that $\theta(T) - \theta(0) = 2\pi n$.

In terms of the single-valued vector $|\tilde{\psi}\rangle$ the cyclic state $|\psi\rangle$ is represented as:

$$|\psi(t)\rangle = e^{i\varphi(t)}|\tilde{\psi}(t)\rangle\tag{12}$$

and the Schrödinger equation gives:

$$\varphi(T) = \delta + \gamma = -\int_0^T \langle\tilde{\psi}|H|\tilde{\psi}\rangle + i\int_{\mathcal{C}}\langle\tilde{\psi}|d|\tilde{\psi}\rangle.\tag{13}$$

The first term in Eq. (13) is naturally identified with the non adiabatic dynamical phase and depends explicitly on the detailed structure of the Hamiltonian. One can show [3] that in the adiabatic limit it reduces to the expression $\delta = -\int E_n$ mentioned above. Note that the dynamical phase may also be calculated as $\delta = -\int\langle\psi|H|\psi\rangle$ since the phase difference between $|\psi\rangle$ and $|\tilde{\psi}\rangle$ drops out in the matrix element. The second term is a non adiabatic generalization of the geometrical Berry phase (9) and it depends only on the closed path $\mathcal{C} \in \mathbb{C}P^N$ spanned by $|\tilde{\psi}\rangle$ during its cyclic evolution. Indeed, the geometrical phase γ' calculated by means of any other single-valued vector $|\tilde{\psi}'\rangle = e^{i\theta}|\tilde{\psi}\rangle$ (see the note after Eq. (11)) differs from γ, Eq. (13):

$$\gamma' = i\int_{\mathcal{C}}\langle\tilde{\psi}|e^{-i\theta}\,d\,e^{i\theta}|\tilde{\psi}\rangle = \gamma - \int_{\mathcal{C}}d\theta = \gamma - 2\pi n\tag{14}$$

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and therefore the difference $\gamma' - \gamma$ is inessential.

In order to explicitly evaluate the geometrical phase we note that single-valued vectors $|\tilde{\psi}\rangle$ are parameterized by the homogeneous coordinates w_i on $\mathbb{C}P^N$, $|\tilde{\psi}\rangle = \{w_1, ..., w_N\}$, and the scalar product $\langle\tilde{\psi}'|\tilde{\psi}\rangle$ is given by:

$$|\tilde{\psi}\rangle = \{w_1, ..., w_N\}, \qquad |\tilde{\psi}'\rangle = \{w'_1, ..., w'_N\},$$

$$\langle\tilde{\psi}'|\tilde{\psi}\rangle = \exp\{K(\bar{w}', w) - \tfrac{1}{2}K(\bar{w}', w') - \tfrac{1}{2}K(\bar{w}, w)\}, \tag{15}$$

where $\bar{w}$ denotes complex conjugation and $K(\bar{w}', w)$ is the Kähler potential on $\mathbb{C}P^N$. One obtains [13, 17]:

$$\gamma = -\int_{\mathcal{C}} \mathrm{Im}\left[\frac{\partial K(\bar{w}, w)}{\partial w} dw\right] = \int_{\mathcal{C}} \frac{i}{2}\frac{\bar{w}_i dw_i - w_i d\bar{w}_i}{1 + \bar{w}_i w_i}. \tag{16}$$

The appearance of a non trivial geometrical phase during a cyclic evolution of the quantum mechanical system is guaranteed then by the following topological arguments [5, 6, 14]. Let variables λ_i parameterize a manifold Λ. We have shown that the Hamiltonian $H(\lambda)$ defines the mapping $f : \Lambda \to \mathbb{C}P^N$ which maps any curve in Λ to a curve in $\mathbb{C}P^N$. Since $\pi_n(\mathbb{C}P^N) = 0$, $n \neq 2$ and $\pi_2(\mathbb{C}P^N) = \mathrm{Z}$ the mapping f might not be homotopic to zero provided that $\pi_2(\Lambda) \neq 0$. On the other hand, the line bundle (10) is non trivial and therefore the mapping $S^{2N+1} \to \mathbb{C}P^N \xrightarrow{f^*} \Lambda$ defines a non trivial U(1) bundle over Λ. In this case the global definition of the phase of the state vector is impossible over parameter space and this fact guarantees the appearance of the Berry phase. Note that the geometrical phase factor when written in the form:

$$\gamma = i\int_{\mathcal{C}} \langle\tilde{\psi}|d|\tilde{\psi}\rangle = \int_{\mathcal{C}} A = \int_{\mathcal{S}_{\mathcal{C}}} F, \tag{17}$$

where $\mathcal{S}_{\mathcal{C}}$ is an arbitrary surface which bounds the contour $\mathcal{C} \in \Lambda$, defines the winding number density $\frac{1}{2\pi}F \sim \partial_{\lambda_i}\langle\tilde{\psi}| \partial_{\lambda_j}|\tilde{\psi}\rangle d^2\lambda^{ij}$ of the mapping $\Lambda \ni S^2 \to \mathbb{C}P^N$, since $(d\langle\tilde{\psi}|) \wedge (d|\tilde{\psi}\rangle)$ is the first Chern class of the bundle (10). In particular, the integral of $\frac{1}{2\pi}F$ over $S^2 \in \Lambda$ gives the degree of the mapping $\Lambda \ni S^2 \to \mathbb{C}P^N$. This implies in turn that the integral (17) calculated for two different surfaces $\mathcal{S}_{\mathcal{C}}$ and $\mathcal{S}'_{\mathcal{C}}$ may differ only by $2\pi n$ and therefore the phase factor $e^{i\gamma}$ is surface independent. The non triviality of the mapping $\Lambda \ni S^2 \to \mathbb{C}P^N$, together with a line bundle structure (10), is essentially the only reason why the monopole potential repeatedly appears in studies of the Berry phase.

Note that both dynamical and geometrical phase factors calculated in unitary transformed state basis are generally different. Indeed, going to unitary transformed states $|\tilde{\psi}'\rangle = U|\tilde{\psi}\rangle$ is equivalent to going to the Hamiltonian $H'(\lambda) = U^+H(\lambda)U - iU^+\partial_t U$. The corresponding total phase is then:

$$\varphi' = -\int_0^T \langle\tilde{\psi}|U^+HU|\tilde{\psi}\rangle + i\int_{\mathcal{C}}\left[\langle\tilde{\psi}|U^+dU|\tilde{\psi}\rangle + \langle\tilde{\psi}|d|\tilde{\psi}\rangle\right] =$$

$$= -\int_0^T \langle\tilde{\psi}'|H|\tilde{\psi}'\rangle + i\int_{\mathcal{C}'}\langle\tilde{\psi}'|d|\tilde{\psi}'\rangle. \tag{18}$$

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It is clear that both dynamical and geometrical phases calculated with H' differ in general from the phases evaluated with H. Nevertheless, there exists a class of unitary transformations $U(t)$ for which the Berry phases are the same. Namely, if for a unitary $U(t)$ we have $\mathcal{C}' = \mathcal{C}$, then $\gamma' = \gamma$ since the geometrical phase depends only on the path $\mathcal{C} \in \mathbb{C}P^N$. It is easy to see that only the U(1) transformations, $U = e^{i\alpha}$ possess this property.

2. Geometrical Phase and Wilson Loop

There is a class of Hamiltonians which are of particular interest and are constructed as follows. The space of state vectors carries a unitary irreducible representation of a compact semisimple Lie group G. The Hamiltonian depends on a set of parameters $\lambda_i(t)$ and for every t is an element of the Lie algebra of G. Then the evolution operator for the Schrödinger equation (2),

$$V(t) = \mathrm{T} \exp\{-i \int_0^t H\} \tag{19}$$

is given by a path $[0; T] \to G$ in the group space and for any t belongs to the representation of G (in the Eq. (19) T denotes the time-ordering). Therefore, if at $t = 0$ we start with an arbitrary state $|\psi(0)\rangle$, then the state vector at the time t, $|\psi(t)\rangle = V(t)|\psi(0)\rangle$ is a generalized coherent state [8]. It is important that the phase of the state $|\psi(t)\rangle$ is naturally fixed with respect to a specific choice of the coherent state basis $|z\rangle$. Indeed, an arbitrary state vector may be represented as $|\psi(t)\rangle = e^{i\varphi(t)}|z(t)\rangle = e^{i\varphi(t)}U(z(t))|0\rangle$, where the complex variables $\{z_1, ..., z_n\}$ parameterize the coset space G/H, H is the Cartan subgroup of G, $U(z) \in G/H$ and $|0\rangle$ denotes an arbitrary fixed vector in the representation space, which is usually taken to be highest or lowest weight vector. Thus the space of all physical states is nothing else but the coset space G/H. The total phase factor acquired by the cyclic state during the evolution (19) may be calculated by means of Eq. (13) with $|\tilde{\psi}\rangle = |z\rangle$ [17, 18].

The evolution operator (19) is of particular importance in gauge theories. Indeed, consider a Wilson loop $W(T)$ on the contour parameterized by coordinates $x_\mu(t)$, $t \in [0; T]$:

$$W(T) = \mathrm{Pexp}\{i \int_0^T A(t)\, dt\}, \qquad A(t) = A_\mu^a(x(t))\, \dot{x}_\mu(t)\, T^a, \tag{20}$$

where T^a are the generators of the gauge group G in the representation considered and the dot denotes differentiation with respect to t. The P-exponent is defined as a solution of the first order differential equations:

$$(i\partial_t + A)|\psi\rangle = 0, \qquad |\psi(t)\rangle = W(t)|\psi(0)\rangle \tag{21}$$

and therefore the Wilson loop is just the evolution operator (19) with a time-dependent Hamiltonian $H = -A(t)$. Therefore, the discussion above suggests that phases of eigenvectors of the Wilson loop may be naturally decomposed into dynamical and geometrical parts. Note that such a decomposition cannot be gauge invariant. Indeed, we have shown in the previous section that the dynamical and geometrical phases calculated for unitary

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equivalent states vectors are generally different. On the other hand, the transformation $H' = U^+ H U - i U^+ \partial_t U$ is exactly the same as the gauge transformation.

In the simplest case of $G = SU(2)$ and the Wilson loop in the fundamental representation, the procedure is as follows. The space G/H is a two-dimensional sphere which is parameterized by a complex coordinate z. The coherent states are build over the highest weight vector $|0\rangle$, $\sigma^3|0\rangle = |0\rangle$, σ^a are the Pauli matrices:

$$|z\rangle = \frac{1}{\sqrt{1+|z|^2}} e^{z(\sigma^1 - i\sigma^2)/2} |0\rangle. \tag{22}$$

The action of the group element $g \in SU(2)$ in the coherent state basis[2] is [8]:

$$g = \begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{bmatrix}, \qquad g|\zeta\rangle = e^{i\phi(g,\zeta)} |\zeta_g\rangle, \tag{23}$$

$$\phi(g,\zeta) = \arg[\beta\zeta + \alpha], \qquad \zeta_g = \frac{\bar{\alpha}\zeta - \bar{\beta}}{\beta\zeta + \alpha}. \tag{24}$$

The Wilson loop operator, Eq. (20), has two eigenstates $|z_\pm\rangle$:

$$W(t) = \begin{bmatrix} \alpha(t) & \beta(t) \\ -\bar{\beta}(t) & \bar{\alpha}(t) \end{bmatrix}, \qquad W(T)|z_\pm\rangle = e^{\pm i\varphi(T)} |z_\pm\rangle. \tag{25}$$

It is sufficient to consider a particular eigenvector, e.g. $|z_+\rangle$, which tends to $|0\rangle$ when $\beta \to 0$ (the z_- approaches infinity in the limit $\beta \to 0$). The cyclic state $|\zeta(t)\rangle$ and the single valued vector $|z(t)\rangle$ are constructed as follows:

$$|\zeta(t)\rangle = W(t)|z_+\rangle = e^{i\varphi(t)}|z(t)\rangle, \qquad z(t) = \frac{\bar{\alpha}(t)\,z_+ - \bar{\beta}(t)}{\beta(t)\,z_+ + \alpha(t)}. \tag{26}$$

Therefore, the total phase factor $\varphi(T)$ acquired by the cyclic state $|\zeta\rangle$ is

$$\varphi(T) = \arg[\beta(T)z_+ + \alpha(T)] = \delta + \gamma, \tag{27}$$

$$\delta = \int_0^T \langle z|A|z\rangle = \int_0^T \langle \zeta|A|\zeta\rangle, \qquad \gamma = i\int_\mathcal{C} \langle z|d|z\rangle = -\int_\mathcal{C} \mathrm{Im}\left[\frac{\partial K(\bar{z}, z)}{\partial z}dz\right], \tag{28}$$

where $K(\zeta, z) = \ln[1 + \bar{\zeta}z]$ is the Kähler potential on G/H. Similar to the quantum mechanical example, the dynamical phase may be calculated with either the single-valued vector or with the cyclic state. The trace of the Wilson loop operator is evidently $\frac{1}{2}\mathrm{Tr}\,W(T) = \cos\varphi(T)$.

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[2] Note that our notations are slightly different (although equivalent) to that of Ref. [8].

3. Lattice Implementation.

Eqs. (22–28) are directly applicable in the lattice gauge theories (LGT). Consider therefore the Wilson loop operator as it appears on the lattice:

$$W = \begin{bmatrix} \alpha_w & \beta_w \\ -\bar{\beta}_w & \bar{\alpha}_w \end{bmatrix} = g_N \, g_{N-1} \, ... \, g_1 \,, \qquad g_i = \begin{bmatrix} \alpha_i & \beta_i \\ -\bar{\beta}_i & \bar{\alpha}_i \end{bmatrix}. \qquad (29)$$

Its eigenstate is determined by

$$W | z_+ \rangle = e^{i\varphi} | z_+ \rangle \,, \qquad z_+ = \frac{-i}{\beta_w} \left[\, \text{Im}\alpha_w - \text{sign}(\text{Im}\alpha_w)\sqrt{(\text{Im}\alpha_w)^2 + |\beta_w|^2} \, \right] \,, \quad (30)$$

where the eigenvector which obeys the condition $z_+ \to 0$ when $\beta_w \to 0$ has been selected. The calculation of the total phase φ is now straightforward. Indeed, according to Eqs. (23,24) the cyclic $| \zeta_k \rangle$ and single-valued $| z_k \rangle$ vectors are given by:

$$\begin{aligned} g_k \, | \zeta_{k-1} \rangle &= | \zeta_k \rangle \\ g_k \, | z_{k-1} \rangle &= | z_k \rangle \, e^{i\varphi_k} \\ k = 1, ..., N \,, \qquad z_0 &= \zeta_0 = z_N = \zeta_N e^{-i\varphi} = z_+ \,. \end{aligned} \qquad (31)$$

Therefore, the total phase factor φ of the Wilson loop is calculated as:

$$\varphi = \sum_{k=1}^{N} \varphi_k \,. \qquad (32)$$

By construction, the total phase φ satisfies $\frac{1}{2} \text{Tr} \, W = \cos\varphi$. Moreover, Eq. (32) reduces to the well known Abelian expression when all g_i are diagonal.

The decomposition of the total phase (32) into the dynamical and geometrical parts is more subtle. The point is that such a decomposition is not possible without a particular parameterization of the Wilson loop W and of the matrices g_k in terms of a continuous time variable. We choose a natural parameterization:

$$W(t) = \begin{cases} g_1(t) \,, & 0 \le t < 1 \\ ... \\ g_k(t)g_{k-1}...g_1 \,, & k - 1 \le t < k \\ ... \\ g_N(t)...g_1 \,, & N - 1 \le t < N \end{cases} \,, \qquad (33)$$

where the time-dependent matrices $g_k(t)$ are defined as follows: if $g_k = \exp\{iA_k\}$ then $g_k(t) = \exp\{iA_k(t - k + 1)\}$. With the parameterization (33), the Wilson loop $W(t)$ describes a continuous path in the SU(2) group manifold and the cyclic state $| \zeta(t) \rangle = W(t)| z_+ \rangle$ satisfies the Schrödinger equation $i\partial_t | \zeta \rangle = -A_k| \zeta \rangle$, $k-1 \le t < k$, $k = 1, ..., N$. Since $g_k(t)A_k = A_k g_k(t)$, we obtain from Eq. (28) the dynamical phase:

$$\delta = \sum_{k=1}^{N} \langle z_{k-1} | A_k | z_{k-1} \rangle = \sum_{k=1}^{N} \langle \zeta_{k-1} | A_k | \zeta_{k-1} \rangle \,, \qquad (34)$$

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where $| z_k \rangle$ are defined by Eq. (31) and the matrix elements $\langle z | A | z \rangle$, $A = A^a \sigma^a / 2$ are given by

$$\langle z | A | z \rangle = \frac{1}{1 + |z|^2} \left[\frac{1}{2} A^3 (1 - |z|^2) + \text{Re}(z A^-) \right], \qquad A^- = A^1 - i A^2. \quad (35)$$

Moreover, the Berry phase associated with the Wilson loop (29) is $\gamma = \varphi - \delta$.

Consider the Wilson loop which is the ordered product of the link matrices $g_{x,\mu}$ taken along the boundary of the elementary plaquette situated at the point x and directed in the $\{\mu, \nu\}$ plane:

$$W = g^+_{x,\nu} g^+_{x+\nu,\mu} g_{x+\mu,\nu} g_{x,\mu}. \quad (36)$$

Let the area of the plaquette be $\sigma^{\mu\nu}$, then the dynamical phase $\delta_{x,\mu\nu} \, \sigma^{\mu\nu}$ is:

$$\delta_{x,\mu\nu} \, \sigma^{\mu\nu} = \langle z_+ | \, A_{x,\mu} + g^+_{x,\mu} A_{x+\mu,\nu} g_{x,\mu} - g^+_{x,\mu} g^+_{x+\mu,\nu} A_{x+\nu,\mu} g_{x+\mu,\nu} g_{x,\mu} -$$
$$- g^+_{x,\mu} g^+_{x+\mu,\nu} g_{x+\nu,\mu} A_{x,\nu} g^+_{x+\nu,\mu} g_{x+\mu,\nu} g_{x,\mu} \, | z_+ \rangle \, \sigma^{\mu\nu}. \quad (37)$$

Note that the state $| z_+ \rangle$ depends on the plaquette considered. For an infinitesimal Wilson loop Eq. (37) can be expanded in powers of $\sigma^{\mu\nu}$. In the leading order the dynamical phase is given by:

$$\delta_{x,\mu\nu} \, \sigma^{\mu\nu} = \langle z_+ | F_{\mu\nu} - i[A_\mu, A_\nu] | z_+ \rangle \, \sigma^{\mu\nu} = \langle z_+ | D_{[\mu} A_{\nu]} | z_+ \rangle \, \sigma^{\mu\nu}, \quad (38)$$

where $F_{\mu\nu}$ and D_μ are the continuum field strength tensor and covariant derivative, respectively:

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]} - i[A_\mu, A_\nu] = \frac{1}{2} \sigma^a \left(\partial_{[\mu} A^a_{\nu]} + \varepsilon^{abc} A^b_\mu A^c_\nu \right), \qquad D_\mu = \partial_\mu - i[A_\mu, \ ]. \quad (39)$$

It can be checked directly that the total phase factor is $\varphi_{x,\mu\nu} \, \sigma^{\mu\nu} = \langle z_+ | F_{\mu\nu} | z_+ \rangle \, \sigma^{\mu\nu}$. Therefore, the Berry phase for the infinitesimal Wilson loop is given by:

$$\gamma_{x,\mu\nu} \, \sigma^{\mu\nu} = i \langle z_+ | [A_\mu, A_\nu] | z_+ \rangle \, \sigma^{\mu\nu}. \quad (40)$$

As expected from the quantum mechanical example above, the value of the Berry phase measures to what extent the gauge fields are non-Abelian. Indeed, for pure Abelian potentials $A^a_\mu = \delta^{a,3} A_\mu$ the Berry phase vanishes and the total phase of the infinitesimal Wilson loop is given by the dynamical phase alone:

$$\varphi_{x,\mu\nu} = \delta_{x,\mu\nu} = \frac{1}{2} \partial_{[\mu} A_{\nu]}. \quad (41)$$

As is mentioned above, the decomposition of the total phase of the Wilson loop into the dynamical and geometrical parts, Eqs. (28, 34, 38, 40), is gauge dependent. Nevertheless, it is possible to find a subgroup of the gauge group which does not change the Berry phase. Indeed, the defining property of these gauge transformations is the same as in the quantum mechanics, namely, they should not change the path spanned by the state

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vector $|z(t)\rangle$ in $G/H = SU(2)/U(1)$. It is clear that the subgroup in question is a U(1) subgroup of SU(2). Since $1/2\,\text{Tr}\,W = \cos\varphi$, the quantity $\varphi \bmod 2\pi$ is gauge invariant and the effect of the U(1) gauge transformations which do not change the Berry phase, is to shift the dynamical or, equivalently, total phase by $2\pi k$, $k \in Z$. It is amusing to note that with $k \neq 0$ these are indeed "large" U(1) gauge transformations familiar from the example of the U(1) compact electrodynamics.

Moreover, when the gauge is fixed the considerations of the Section 1 are applicable. Namely, any closed path $\mathcal{C}$ in the physical space M is naturally mapped into a closed path in G/H, the relevant mapping being determined by the Wilson loop operator, calculated on $\mathcal{C}$. The degree of the mapping $f : \mathcal{S} \to G/H$, where $\mathcal{S}$ is a closed two-dimensional surface in M, is given by:

$$\deg[f] = \frac{1}{2\pi} \int_{\mathcal{S}} \gamma = \frac{1}{2\pi} \int_{G/H} F \,, \tag{42}$$

where F is the first Chern class of the non trivial Hopf bundle $G \to G/H$. Note that when $\deg[f] \neq 0$ the Berry phase γ contains a string-like singularity (Dirac string) somewhere on the surface $\mathcal{S}$. As noted above, the position of the singularity may be arbitrary shifted by U(1) gauge transformations.

Let the surface $\mathcal{S}$ be parameterized by coordinates $\tilde{x}(\sigma)$, $\sigma = \{\sigma^\alpha\} = \{\sigma^1, \sigma^2\}$. Using Eqs. (30, 35) it is straightforward to show that:

$$\int_{\mathcal{S}} \gamma = -\frac{1}{2} \int_{\mathcal{S}} \varepsilon^{abc}\, n^a(\sigma)\, A^b_\mu(\tilde{x})\, A^c_\nu(\tilde{x})\, \mathrm{d}^2\sigma^{\mu\nu} \qquad \mathrm{d}^2\sigma_{\mu\nu} = \varepsilon^{\alpha\beta}\partial_\alpha\tilde{x}_\mu\partial_\beta\tilde{x}_\nu \,, \tag{43}$$

where the world-sheet vector field $n^a(\sigma)$, $\vec{n}^2 = 1$ was introduced in Ref. [10]:

$$n^a(\sigma) = (\mathrm{d}^2\sigma \cdot F^a)\left[(\mathrm{d}^2\sigma \cdot F^b)^2\right]^{-1/2} \,, \qquad (\mathrm{d}^2\sigma \cdot F^a) = \mathrm{d}^2\sigma^{\mu\nu} F^a_{\mu\nu} \,. \tag{44}$$

Note, however, that $n^a(\sigma)$, Eq. (44), is build on the $F^a_{\mu\nu}$, not $^*F^a_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F^a_{\lambda\rho}$ as in Ref. [10].

4. Numerical Results.

In this section we show that the formalism presented above may be used in lattice simulations and present the numerical results which concern the total phase, Eq. (32). The total phase of the Wilson loop is of particular importance since it naturally includes the contribution of both Abelian-like and non-Abelian fields. As for the lattice implementation of the dynamical and Berry phases, we have not considered it yet although it seems straightforward.

The trace of the Wilson loop, calculated over the boundary of an elementary plaquette p, is gauge invariant and is proportional to $\cos\varphi_p$. Therefore, the total phase φ may be represented in terms of the gauge invariant quantity $\varphi_p \bmod 2\pi$ and an integer number k_{*p}:

$$\varphi_p = \varphi_p \bmod 2\pi + 2\pi k_{*p} \,, \tag{45}$$

where $^*p$ is the plaquette dual to p. By analogy with the corresponding equality in the compact U(1) LGT the integer k_{*p} counts the number of Dirac strings piercing the plaquette p. A set of numbers k_{*p} is an integer valued 2–form $k \in {}^*C_2(Z)$, which belongs to oriented plaquettes on the dual lattice. As usual, we define the monopoles as end points of the Dirac strings (45):

$$j_{x,\mu} = (\delta k)_{x,\mu}, \qquad j \in {}^*C_1(Z). \qquad (46)$$

The monopole current $j_{x,\mu}$ is an integer valued 1–form on the dual lattice. While all the definitions above are unique in terms of the potentials (or link matrices), the potentials themselves are gauge dependent. As a result, the values of k_{*p} are gauge dependent as well and at first sight the definition (46) is devoid of any physical meaning. Clearly, the issue of the gauge dependence is crucial for physical applications of the procedure developed above.

Therefore we introduce at this point a physically motivated gauge following the logic similar to that of the paper [11]. The observation is that in the continuum limit both the Dirac strings and monopoles correspond to singular gauge potentials A. It is easy to imagine, therefore, that by going to arbitrary large A, so to say inflated by the gauge transformations, one can generate an arbitrary number of spurious strings and monopoles. On the other hand, by minimizing potentials one may hope to squeeze the number of the topological defects to its minimum and these topological defects may have physical significance. In order to quantify what is understood by minimizing the potentials, consider the Lorenz gauge which is defined by the requirement that the functional

$$R = \sum_{x,\mu} (1 - 1/2 \operatorname{Tr} g_{x,\mu}) \qquad (47)$$

is minimal on the gauge orbit. In the naive continuum limit Eq. (47) reduces to $R = 1/4 \int (A_\mu^a)^2$. Thus, the Lorenz gauge is singled out since in this gauge the link matrices are as close to unity as possible and the gauge dependent singularities are suppressed as much as possible. As we shall see in a moment, this heuristic justification of the gauge choice can be checked a posteriori.

We have performed numerical experiments with monopoles defined by Eq. (46) in the Lorenz gauge. The calculations were done on the 12^4 and 4×12^3 lattices with periodic boundary conditions using 30 well equilibrated and statistically independent configurations. We used the local over-relaxation algorithm to fix the Lorenz gauge. The gauge was considered fixed when at each site the gauge transformation matrix Ω_x, which locally maximizes Eq. (47), satisfy $1 - 1/2 \operatorname{Tr} \Omega_x < 10^{-6}$. In order to circumvent the Gribov copies problem, each thermalized configuration was randomly gauge transformed to five gauge equivalent configurations and the Lorenz gauge was fixed again on each gauge copy. The measurements were performed on the configuration for which the functional (47) is minimal.

The density of monopoles, Eq. (46), is given by:

$$\rho = \frac{1}{4V} \sum_{x,\mu} |j_{x,\mu}|, \qquad (48)$$

where V is the lattice volume. We have measured the density ρ in the Lorenz gauge as a function of the bare coupling $\beta = 4/g^2$. The logarithm of ρ versus β on the symmetric 12^4 lattice is shown on the Fig. 1a. The solid curve on the figure is the renormalization group prediction:

$$\rho \;\; = \;\; \text{const.} \; \cdot \; \beta^{153/121} \; \exp\left\{ -\frac{9\pi^2}{11}\beta \right\} , \qquad (49)$$

plotted in the logarithmic scale. As is clear from the plot, the density ρ perfectly scales towards the continuum limit. Thus the monopoles (46) are physical objects and this justifies the choice of the Lorenz gauge, motivated above. Note that the finite physical density of the monopoles (46) in the continuum gluodynamics implies in particular, that even in the Lorenz gauge the gauge fields are rather singular. The relevance of the singular fields in QCD is discussed in Ref. [19].

It is interesting to consider also the behavior of the density (48) across the deconfinement phase transition. On the Fig. 1b we plot the logarithm of ρ on the 4 x 12^3 lattice, the solid curve is the scaling law (49). The monopole density jumps at the critical coupling and remains non zero in the deconfinement phase. Note that after the phase transition ρ also seems to scale correctly towards the continuum limit, although this question needs further investigations.

Another interesting quantity related to the monopole dynamics is the asymmetry of the monopole currents [20]:

$$A \;\; = \;\; \frac{1}{3} < \sum_{x,\mu=1,2,3} |j_{x,\mu}| > / < \sum_{x} |j_{x,0}| > , \qquad (50)$$

which is known to be the order parameter of the deconfinement phase transition when the monopoles are defined in the Maximal Abelian gauge. Indeed, the dual superconductor confinement scenario suggests that the Abelian monopoles are condensed in the confinement phase, while they are almost static at high temperatures. Therefore, the asymmetry of Abelian monopole currents is unity at zero temperature and vanishes in the deconfinement phase with rising temperature. Since the definition of the monopoles (46) refers to the Lorenz gauge, the behavior of the asymmetry (50) across the phase transition is worth to be considered anew.

The Fig. 2 represents the asymmetry A versus β on the 12^4 (diamonds) and 4 x 12^3 (squares) lattices. As expected, the asymmetry measured on the symmetric lattice is unity within the numerical errors. On the asymmetric lattice the asymmetry shows a rapid jump at the critical coupling, being substantially smaller after the phase transition. Thus, in the deconfinement phase the monopoles defined in the Lorenz gauge, Eq. (46), are mostly static. Note that the asymmetry needs not to be unity at $\beta < \beta_{crit}$ on the asymmetric lattice since the Lorenz invariance is explicitly broken.

In order to clarify the physical relevance of the monopoles constructed consider the correlation of the local $SU(2)$ action density and the monopole currents [12]. In particular, we define the relative excess of the action density associated with the monopole as follows:

$$\eta \;\; = \;\; \frac{S_m \; - \; S}{S} , \qquad (51)$$

where S denotes the average vacuum action density $S = <1 - 1/2\,\mathrm{Tr}\,U_p>$. The quantity S_m is the average value of $1 - 1/2\,\mathrm{Tr}\,U_p$ calculated on the plaquettes, which belong to the 3-dimensional cubes $C_{x,\mu}$ dual to the monopole currents $j_{x,\mu}$:

$$S_m \;=\; < \frac{1}{6} \sum_{p \in \partial C_{x,\mu}} \left(1 - \frac{1}{2}\,\mathrm{Tr}\,U_p\right) > \;.\tag{52}$$

The average is implied over all cubes $C_{x,\mu}$ for which $j_{x,\mu} \neq 0$. For static monopoles only the magnetic part of the SU(2) action density contributes to S_m. We have measured the quantity η on the symmetric 12^4 lattice, the result is plotted on the Fig. 3. It is clearly seen that the monopoles in the Lorenz gauge are indeed locally correlated with non-Abelian SU(2) action density.

Such an excess of the action associated with monopoles is quite a common phenomenon. Indeed, monopoles are correlated with instantons [21] which realistically represent non-perturbative background in QCD [22]. A possible general connection between monopoles and background fields was recently pointed out in Ref. [10]. Moreover, the excess of the action associated with monopoles (46) is numerically close to the action excess around the Abelian monopoles in the Maximal Abelian gauge [12].

The latter observation rises the question about the local correlation between these objects. A priori, it is by no means evident that the monopoles defined by the Eq. (46) and the Abelian monopoles in the Maximal Abelian gauge should be correlated. Indeed, the monopoles in the Maximal Abelian gauge are by construction Abelian objects, the field of which have a natural electromagnetic interpretation. In particular, in order to explain the confinement phenomenon as due to these Abelian monopoles, they should be condensed in the vacuum of gluodynamics. Contrary to that, the Dirac strings and monopoles, Eqs. (45,46), are non-Abelian field configurations. There is no simple electromagnetic-like interpretation of them and therefore their relation to the confinement is a separate question which should be considered anew. In particular, the contribution of the field configurations which correspond the monopoles (46) to the expectation value of the Wilson loop is unknown. Moreover, one cannot say that if the monopoles (46) are related to the confinement, they should be condensed in the vacuum. Therefore, the correlation of the Abelian monopoles in the Maximal Abelian gauge and the monopoles defined by Eq. (46) is an interesting question, which we leave for future investigation.

Due to the non-Abelian nature of the strings (45) it is not evident a priori that they are unphysical. Indeed, in case of Abelian Dirac strings the U(1) gauge freedom guarantees that no physical result depends on the string position. This is true in particular in Abelian gauges of gluodynamics. Contrary to that, the strings (45) are considered in the Lorenz gauge in which there is no remaining U(1) gauge symmetry. Therefore the physical irrelevance of the strings (45) should be checked separately.

We have numerically found that, indeed, the strings (45) are unphysical objects. For example, their density, defined analogously to the Eq. (48), scales at weak coupling according to the Eq. (49), whereas the density of physically relevant string-like excitations behaves as $\rho_{\mathrm{string}} \sim \exp\{-\frac{6\pi^2}{11}\beta + \frac{102}{121}\ln\beta\}$. Furthermore, the action density calculated

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around the string away from its boundary is the same as the vacuum action density within the numerical errors. Thus the strings (45) are similar to the Dirac strings in the compact electrodynamics. The work is currently in progress to find the rigorous explanation of this fact.

Conclusions.

We have considered the well known quantum mechanical notion of the geometrical Berry phase in the context of non Abelian gauge theories. The bridge between the Berry phase and non-Abelian gauge theories is provided by the Wilson loop which can be considered as a quantum mechanical evolution operator. The role of the corresponding time-dependent Hamiltonian is played then by the gauge potential $A(t)$. This observation allows to utilize in gauge theories results obtained earlier for the Berry phase in case of Hamiltonians belonging to a Lie algebra of group G. In particular, it is known that the space of physically distinct states in this case is the coset space G/H, H being the Cartan subgroup of G. Moreover, the evolution of this quantum mechanical system is naturally described by the generalized coherent states.

We have applied these ideas to the Wilson loop operator in the SU(2) gluodynamics both in the continuum and lattice formulations. In particular, we have presented an explicit construction of the phase φ of the Wilson loop, which is not restricted to the interval $[-\pi; \pi]$, but still satisfies $1/2 \operatorname{Tr} W = \cos \varphi$. We have also shown that in the coherent states basis the phase φ naturally decomposes into dynamical and Berry phases and discussed the topological construction inherent to the Berry phase in gluodynamics. Our considerations, when applied to an elementary plaquette, give a lattice analog of the Dirac strings and monopoles. The U(1) group which is usually associated with monopole-like defects is now defined locally in terms of the infinitesimal Wilson loops, or elementary plaquettes.

The division of the total phase into the dynamical and geometric phases is gauge dependent. In case of quantum mechanics such a decomposition also depends on a particular choice of the basis. We argue, therefore, that for physical applications one has to consider a gauge in which the gauge fields are as smooth as possible, thus suppressing all gauge dependent singularities. Following the logic similar to that of the paper in Ref. [11] we identify this gauge as the Lorenz gauge.

We have studied the dynamics of the newly defined monopoles in numerical simulations on the lattice and presented a strong evidences that these objects are relevant in the continuum limit. Since in the continuum limit the field configurations which produce these monopoles become singular, our findings support for the idea that even in the Lorenz gauge, in which the gauge potentials are as smooth as possible, the singular fields are important. More generally, our results provide with a measure of singular fields in the vacuum. Phenomenology related to such fields in the continuum gluodynamics has been discussed in Ref. [19]

It is worth emphasizing that the construction we have presented is essentially non-Abelian. Only in the particular case when all the gauge fields have the same color ori-

entation, it reduces to the known construction of Dirac strings and monopoles in the compact U(1) gauge model. Therefore, the relevance of our considerations to the confinement requires further investigation. It is also important to study the relation, if any, between monopoles we have constructed and other topological defects, for example, the Abelian monopoles in the Maximal Abelian gauge of the gluodynamics.

Acknowledgments.

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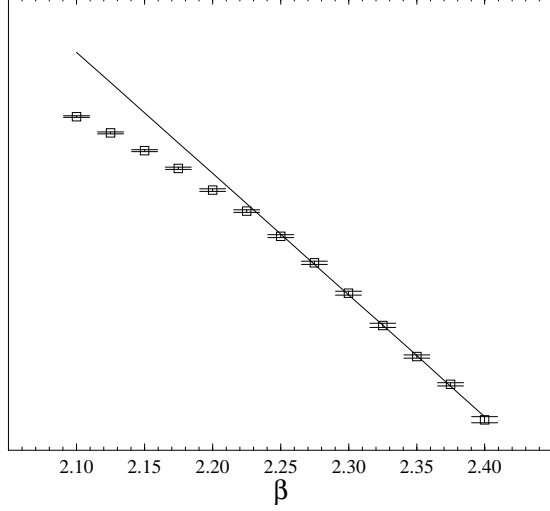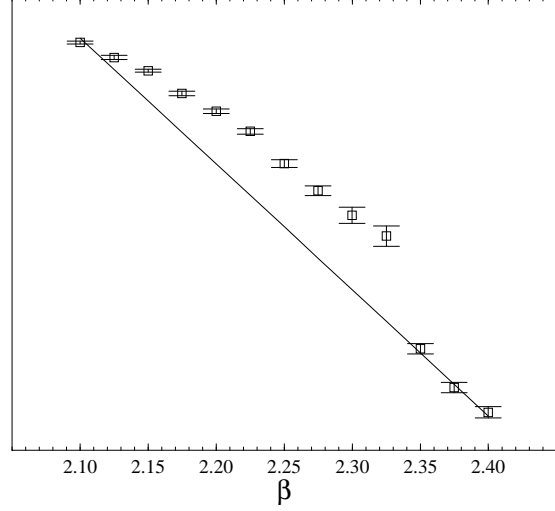
(a)   (b)

Figure 1: The logarithm of the monopole density, Eq. (48), as a function of $\beta = 4/g^2$. (a) – the symmetric 12^4 lattice; (b) – the asymmetric 4×12^3 lattice. Solid curve represents the renormalization group prediction, Eq. (49).
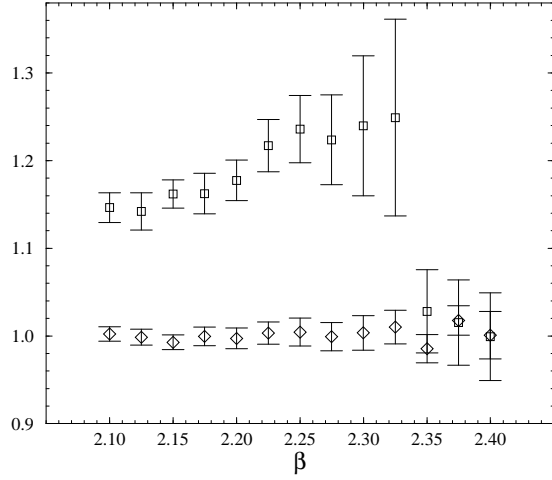

Figure 2: The asymetry of the monopole currents, Eq. (50), versus β on the 12^4 (diamonds) and 4×12^3 (squares) lattices.
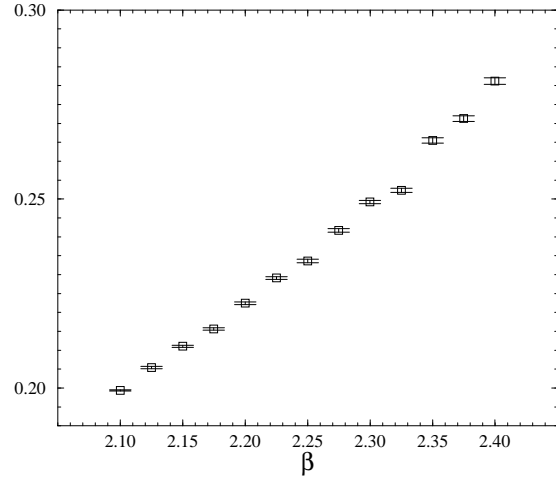

Figure 3: The relative access of the magnetic action density, Eq. (51), around the monopole currents, Eq. (46), for the 12^4 lattice.